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Bistable Behaviour of Light Waves in a Graded-Index Planar Waveguide with Nonlinear Substrate

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Phil. Trans. R. Soc. Lond. A 1984 **313**, 361-362
doi: 10.1098/rsta.1984.0119

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Printed in Great Britain

Bistable behaviour of light waves in a graded-index planar waveguide with nonlinear substrate

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Bistability of a planar guiding structure with a nonlinear refractive index of the upper substrate and a lower linear substrate with a nonabrupt boundary is studied. The central guiding medium is assumed to be linear.

The condition for the propagation of light along the waveguide (see figure 1) is given in the geometrical optics approximation by the well known dispersion formula

$$\Phi(\beta) = 2Kn_1 d\psi - 2\phi_{R1} - 2\phi_{R2} - 2\pi N = 0, \tag{1}$$

where N is an integer number selecting the mode, ϕ_{R1} and ϕ_{R2} are the phase shifts due to total reflection at the nonlinear interface '1–2', and the linear interface '1–3' respectively, and the term $2Kn_1 d\psi$ corresponds to the optical-path phase change in the ray representation and in the small-angle approximation, ψ is the angle between the light ray and $\hat{x}$-axis and d is the waveguide thickness. For a semi-infinite substrate, and in the plane-wave approximation for the guided modes, the nonlinear phase shift ϕ_{R1} can be written (Kaplan 1977) as

$$\phi_{R1} = \mp 2 \arccos |[\Delta\epsilon/\epsilon_1 \pm \{(\Delta\epsilon/\epsilon_1)^2 - 8\psi^2\epsilon_{n1}|E_0|^2/\epsilon_1\}^{\frac{1}{2}}]^{\frac{1}{2}} (4\epsilon_{n1}|E_0|^2/\epsilon_1)^{-\frac{1}{2}}|, \tag{2}$$

where
$$\epsilon_2 = \epsilon_1 + \Delta\epsilon + \epsilon_{n1}|E_0|^2,$$

and $|E_0|^2$ corresponds to the incident intensity at the interface. In our example $|E_0|^2$ is the intensity of the mode in medium 1.

Instabilities for $\Phi(\beta)$ can be found as a function of the incident intensity, in the small angle approximation, when $d\Phi(\beta)/dI_0 = \infty$. After a short calculation it can be shown that instabilities occur at the following two values of incident intensity:

$$I_{0,1} = \frac{\Delta\epsilon}{2\epsilon_{n1}}\left\{1 - \left(\frac{\psi}{\psi_c}\right)^2\right\} \tag{3a}$$

and
$$I_{0,2} = \frac{\Delta\epsilon}{8\epsilon_{n1}}\left(\frac{\psi_c}{\psi}\right)^2, \tag{3b}$$

where $\psi_c^2 \sim \Delta\epsilon/\epsilon_1$ is the critical angle for the total reflection at the boundary 1–2, and we have assumed $\psi < \psi_c$ (guided modes). The instability points correspond to a jump in the phase shift ϕ_{R1}, which makes $\Phi(\beta)$ change from the value $\Phi(\beta) = 0$, corresponding to a guiding structure, to a value $\Phi(\beta) \neq 0$, in which no stable mode is supported. When increasing the input intensity I_0 from a low value for which $\Phi(\beta) = 0$ and a mode is guided in the structure, as soon as $I_0 = I_{0,2}$ we have $\Phi(\beta) \neq 0$ and no mode is propagated any more. When reducing the intensity, at $I_0 = I_{0,1}$ the equality $\Phi(\beta) = 0$ is satisfied again and the structure once more supports a guided mode.

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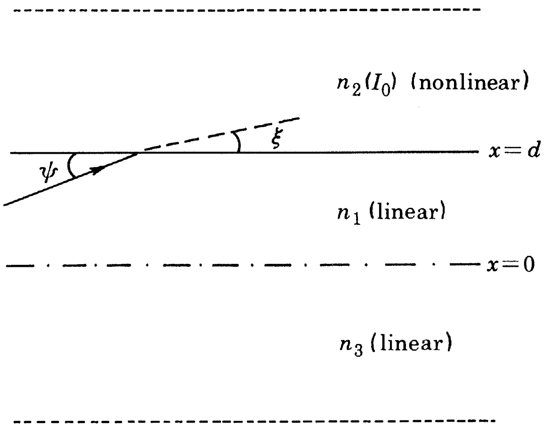

FIGURE 1. Graded-index waveguide with nonlinear substrate, where $n_2(I_0 = 0) < n_1, n_2(I_0 = 0) \geqslant n_3, n_1 > n_3$.

The intensity values at which instabilities occur (equation (3)) are the same as those of Kaplan for bistability of reflection at the single nonlinear interface; so in the approximation used a wave guiding structure has the same bistable behaviour as a nonlinear interface.

The feedback mechanism needed to create hysteresis is provided here by the evanescent field in the nonlinear medium that reduces the critical angle value and affects the phase shift between incident and reflected beams, so the reduced critical angle further reduces the evanescent field. A more complete analysis with nonlinear Maxwell equations confirms this behaviour.

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